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# Engineering Notes

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## Computation of Worst-Case Pilot Inputs for Nonlinear Flight Control System Analysis

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### I. Introduction

MODERN high-performance aircraft are often designed to be naturally unstable (or to have reduced natural stability margins) for performance reasons, such as to improve maneuvrability or to decrease drag and fuel consumption. Such aircraft can, therefore, only be flown by means of a flight control law, which provides artificial stability, and is, hence, a safety-critical system. Two particular difficulties faced by flight control law designers are nonlinearity and uncertainty in the aircraft dynamics. At high angles of attack (AOA) or at high rotation rates, aircraft flight dynamics become highly nonlinear, due to significant levels of cross coupling between axes. Also, all aircraft have significant nonlinearities, associated with limitations in the movement of aerodynamic control surfaces, which can sometimes be excited by large pilot input demands. Significant levels of uncertainty are also inevitably present in even the most detailed aircraft simulation model, so that a large number of uncertain parameters will be used to model variations in mass, inertia, and center of gravity positions; aerodynamic tolerances; air data system tolerances; structural modes; failure cases; etc. For example, in Ref. 1, a good account of the aerodynamic uncertainties and off-nominal design points considered in the space shuttle flight control system verification process is provided.

Because of the described issues, all flight control laws are required to undergo a rigorous certification (or clearance) process before being evaluated in flight tests. The search for "worst-case" pilot control inputs is an important part of this process. For highly agile combat aircraft, a key consideration is the identification of so-called departure susceptibility, the computation of pilot inputs that will excite the nonlinear aircraft dynamics to such an extent as to lead to loss of stability and/or controllability. For flight control laws equipped with a maneuver load limiter, on the other hand, pilot inputs that test the robust functionality of the envelope protection system are required to be computed.

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A number of recent studies have considered the problem. In Ref. 2, the departure susceptibility of the X-31 aircraft was evaluated. Genetic algorithms (GAs) and a high-fidelity nonlinear simulation model were used to search for pilot inputs that maximized a cost function associated with aircraft departures: the absolute sum of certain states of the system, such as attitude rates, AOA, and sideslip angle. The methodology was further developed in Ref. 3 using a multimodal genetic search approach with a refined energy-like cost function, and applied to a full nonlinear simulation model of the Indian Light Combat Aircraft.[4] In neither of these studies, however, was any form of uncertainty considered in the aircraft simulation model. A different, but related, approach to the same problem is reported in Refs. 5 and 6. In these studies, a particular sequence of pilot inputs called the Clonk maneuver, which was developed by SAAB using piloted simulation testing to detect possible departure scenarios for the Gripen aircraft, was applied to the aerodata model in a research environment (ADMIRE)[7] simulation model. Global optimization methods were then used to compute the worst-case combination of uncertain parameters for this sequence of control inputs. In this paper, a global optimization method is used to search simultaneously for the worst-case pilot inputs and the worst-case uncertain parameter combination for the ADMIRE aircraft model. Following Refs. 5 and 6, the cost function used in this study is the maximum value of AOA over a finite time period.

### II. ADMIRE Aircraft Model

The aircraft model considered in the present study is the ADMIRE, a nonlinear six-degree-of-freedom simulation model,[7] developed by the Swedish Aeronautical Research Institute using aerodata obtained from a generic single-seated, single-engine fighter aircraft with a delta-canard configuration. ADMIRE is augmented with a full authority industrial standard flight control system and includes full engine dynamics and actuator models. The model includes a large number of uncertain aerodynamic, actuator, sensor, and inertia parameters, whose values, within specified ranges, can be set by the user. The aircraft dynamics are modeled as a set of 12 first-order coupled nonlinear differential equations and given as $\dot{\boldsymbol{x}}(t) = f[\boldsymbol{x}(t), \boldsymbol{u}(t), \Delta]$; $\boldsymbol{y}(t) = h[\boldsymbol{x}(t), \boldsymbol{u}(t)]$, where $\boldsymbol{x}(t)$ is the state vector with 12 components, that is, velocity, AOA, sideslip angle, and angular rate, attitude, and position vectors. $\Delta$ represents the uncertain parameters in the aircraft simulation model $\boldsymbol{y}(t)$ is the output vector, and $\boldsymbol{u}(t)$ is the control input vector, whose components are left and right canard, left and right inboard/outboard elevon, leading-edge flap and rudder deflection angles, landing gear status (extract/retract), and vertical and horizontal thrust vectoring. The control input is determined by $\boldsymbol{u}(t) = g[\boldsymbol{x}(t), \boldsymbol{y}_{\mathrm{ref}}(t)]$, where $g(\cdot,\cdot)$ is the flight control law, which is provided with the ADMIRE model, and $\boldsymbol{y}_{\mathrm{ref}}(t)$ is the reference demand consisting of the pilot inputs. The present study considers pitch and roll stick inputs only, and the amplitudes of the pilot demands are limited to $\pm 40$ N. Table 1 gives details of the uncertain parameters considered in this study.

The augmented ADMIRE operational flight envelope is defined up to Mach 1.2 and altitude 6000 meters (Ref. 7). The longitudinal control law is gain scheduled over the whole flight envelope with respect to Mach and altitude variations and is designed to ensure robust stability and handling performance over the entire flight envelope. The model also contains actuator rate limiting and saturation blocks, as well as nonlinear stick shaping elements.[8]

**Table 1 Aircraft model uncertain parameters**

| Parameter | Bound | Description |
|---|---|---|
| $\Delta_{\mathrm{mass}}$ | $[-0.1\ \ +0.1]$ | Variation in aircraft mass from nominal one (9100 kg), % |
| $\Delta_{x\,\mathrm{cg}}$ | $[-0.075\ \ +0.075]$ | Variation in position of center of mass, m |
| $\Delta_{C_{m_{\delta_e}}}$ | $[-0.05\ \ +0.05]$ | Uncertainty in pitching moment due to elevator deflection, 1/rad |
| $\Delta_{I_{yy}}$ | $[-0.2\ \ +0.2]$ | Uncertainty in aircraft inertia about $y$ axis from nominal (81,000 kg $\cdot$ m$^2$), % |
| $\Delta_{C_{m_\alpha}}$ | $[-0.05\ \ +0.05]$ | Uncertainty in pitching moment due to AOA, 1/rad |

## III. Global Optimization

The global optimization approach applied in this paper is based on GAs, general purpose stochastic search and optimization procedures that use genetic and evolutionary principles.[9] A recent survey by Fleming and Purshouse[10] provides an overview of recent applications of GAs in the field of control engineering. In Ref. 11, an early flight control application of GAs for the design of lateral autopilot and windshear controller, is reported. Also, in Ref. 12 a recent application of GAs in robust nonlinear control design for hypersonic aircraft is reported.

The basic principle underlying GAs is the assumption that the evolutionary process observed in nature can be simulated on a computer to generate a population of fittest candidate solutions for a given problem. In genetic search techniques, a randomly sourced population of candidates undergoes a repetitive evolutionary process of reproduction through selection for mating according to a fitness function and recombination via crossover with mutation. A complete repetitive sequence of these genetic operations is called a generation. To use this evolutionary method, it is necessary to have a method of encoding each candidate as an artificial chromosome, as well as a means of discriminating between the fitness of different candidates. A fitness function is defined to assign a performance index to each candidate: This function is specific to the problem and is formed from the knowledge domain.

Each optimization variable, or gene, is binary coded according to the required accuracy level and combined sequentially to form the chromosome, which represents a potential candidate solution. The search starts from an initial population consisting of a fixed number of randomly selected candidates. The size of this initial population is given by $N_{\mathrm{size}} = 50$ in the present study. The candidates from the current generation are qualified to produce the successive generations depending on a selection scheme. A roulette wheel selection scheme with a selection probability of 0.6 is applied in this study. During crossover, a recombination operator ensures mixing up of the information content between two different binary coded chromosomes. A single-point crossover with a probability of crossover 0.9 is used here. The point of crossover is determined randomly over the length of bits. Mutation introduces random variations in the population in the search space, by randomly flipping a bit value. The probability of mutation is kept low and fixed at 0.05. The number of maximum generations is the termination criterion and is fixed at 100 generations. The reader is referred to Ref. 9 for more details of GA operators, binary coding schemes, and the theory of genetic search.

## IV. Problem Setup and Analysis

Five different sets of analysis results are presented in this paper.

Analysis 1 is a nominal Clonk analysis. For the nominal simulation model, the pilot control inputs $\boldsymbol{y}_{\mathrm{ref}}(t)$ specified by the Clonk maneuver over the time period $\tau$, $\tau \in [\tau_0\ \ \tau_f]$, are input, and the maximum value of the the chosen cost function (AOA) is computed. This analysis needs only one simulation.

Analysis 2 is an uncertain Clonk analysis. For the uncertain simulation model, for the pilot control inputs $\mathbf{y}_{\mathrm{ref}}(t)$ specified by the Clonk maneuver over the time period $\tau$, $\tau \in [\tau_0\ \ \tau_f]$, the combination of uncertain parameters $\Delta$ that maximizes the chosen cost function (AOA) is computed.

**Table 2 Pilot control input discretization levels and binary representation**

| Pitch stick | Roll stick | Binary levels |
|---|---|---|
| Full maximum [+40 N] | Full maximum [+40 N] | 1 1 |
| Half maximum [+20 N] | Half maximum [+20 N] | 1 0 |
| Half minimum [−20 N] | Half minimum [−20 N] | 0 1 |
| Full minimum [−40 N] | Full minimum [−40 N] | 0 0 |

Analysis 3 has worst-case pilot inputs for the nominal simulation model. For the nominal simulation model, the pilot control inputs $\boldsymbol{y}_{\mathrm{ref}}(t)$ over the time period $\tau$, $\tau \in [\tau_0\ \ \tau_f]$, that maximize the chosen cost function (AOA) are computed.

Analysis 4 has worst-case uncertain parameters for fixed pilot inputs. For the uncertain simulation model, for the worst-case pilot control inputs $\boldsymbol{y}_{\mathrm{ref}}(t)$ computed in analysis 3, the combination of uncertain parameters $\Delta$ that maximizes the chosen cost function (AOA) is computed. This problem can also be interpreted as the computation of the maximum deviation from a nominal trajectory due to uncertainty.

Analysis 5 has worst-case pilot inputs and uncertain parameters. For the uncertain simulation model, the combination of uncertain parameters $\Delta$ and pilot control inputs $\boldsymbol{y}_{\mathrm{ref}}(t)$, over the time period $\tau$, $\tau \in [\tau_0\ \ \tau_f]$, that maximizes the chosen cost function (AOA) is computed simultaneously.

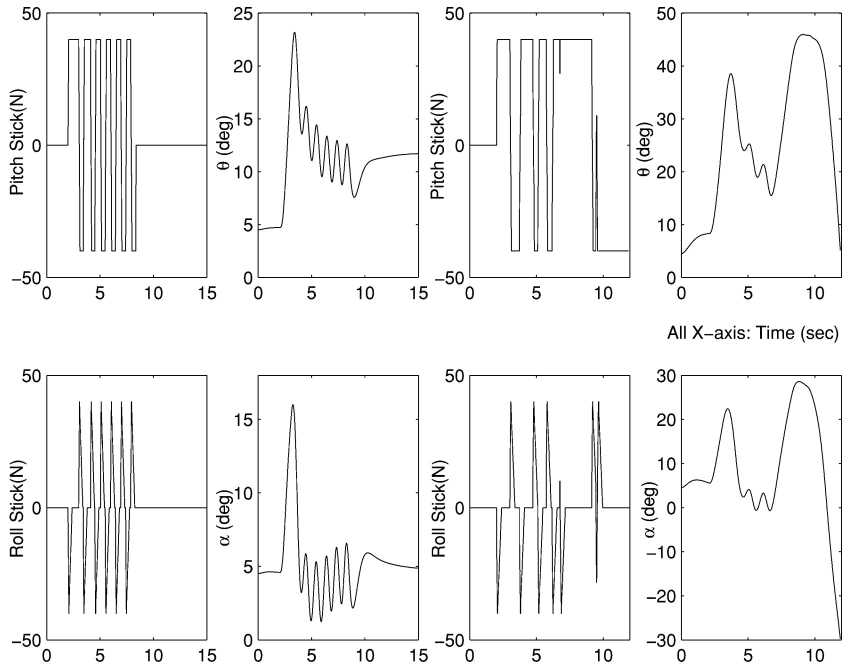
Analyses 1 and 2 are similar to those performed in Refs. 5 and 6, and are given here mainly for the purposes of comparison. For the other three analysis tasks, a common framework consisting of the global optimization algorithm and the closed-loop nonlinear model of the ADMIRE was used. Depending on the type of analysis required, the GA provides the appropriate input to the nonlinear simulation model. When searching for the worst-case pilot control inputs, the search space is discretized into a number of possible amplitude levels as shown in Table 2. This is consistent with current industrial practice, whereby step and doublet inputs of predefined amplitude levels are often used to test flight control laws.[3] The time axis is also discretized, in the sense that changes in the magnitude of pilot inputs are allowed to occur only at regular intervals of 1 and values of pilot inputs are held constant for at least 1 s. A 1-s input frequency is high enough to excite the aircraft's nonlinear dynamics but still low enough to remain a realistic input frequency for a pilot.[2] Because four discretization levels are used for the magnitude of the pilot inputs, in a binary representation 2 bits can represent each pilot input signal over each 1-s interval of time. For each analysis, at the end of the fifth second, all of the control inputs are brought to zero and the simulation is allowed to continue for another 5 s. Each simulation, therefore, runs for a total of 10 s.

## V. Results

All analysis results shown in this paper were generated at a straight and level trimmed flight condition of 0.4 Mach and 3000-m altitude. The computational requirements for the present analyses (apart from analysis 1, which only requires one simulation run) is a maximum of 5000 simulations, which takes approximately 2 h and 45 min on a 3.06-GHz Pentium IV machine running a Windows XP platform.

### Analysis 1

According to the specifications for the Clonk manoeuver (see Refs. 5 and 6), the pilot's pitch stick command switches, with a limited rate, between its maximum magnitude limits when the pitch attitude reaches its maximum or minimum. The roll stick command is simultaneously switched to the opposite extremum to that of the pitch stick command. However, once the roll stick reaches an extremum, it immediately starts moving in the opposite direction at a defined rate, called the roll return rate. The pitch stick command, on the other hand, remains for some additional time on its magnitude limit: This time period is referred to as the pitch stick delay. At the next occurrence of a maximum or minimum of the pitch attitude, the next switching for both the stick commands occurs, and this sequence is then repeated for a specified period of time. Figure 1a




**a) Analysis 1: control input, $\theta$ and $\alpha$ response b) Analysis 2: control input, $\theta$ and $\alpha$ response**

**Fig. 1 Analyses 1 and 2 results.**

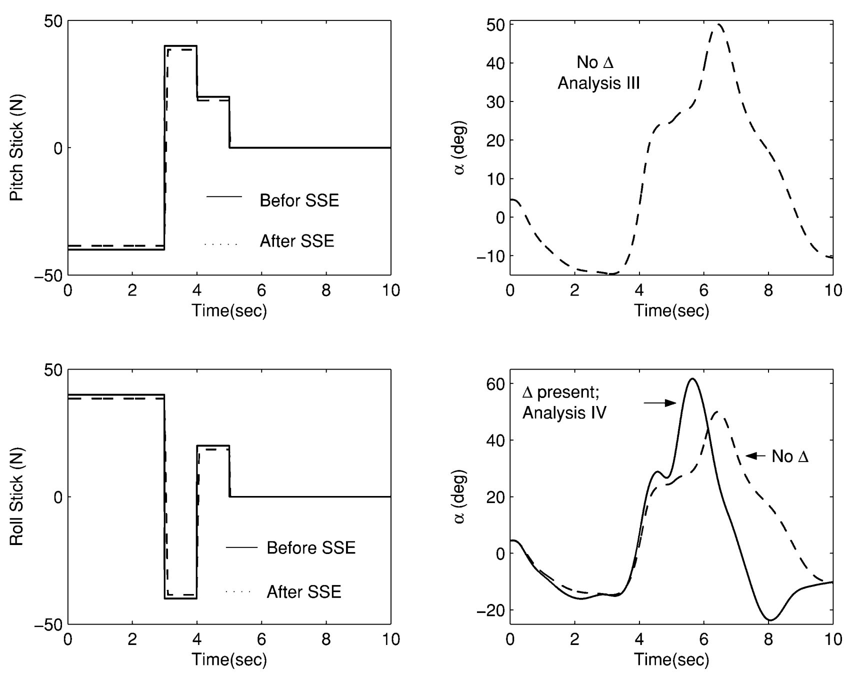



**Fig. 2 Analysis 3 ($\Delta$ not present) and analysis 4 ($\Delta$ present) results.**

shows the pilot input commands generated by the Clonk manoeuver. For the Clonk analysis, a pitch and roll stick deflection rate of 720 and 500 N/s, respectively, and a pitch stick delay and roll return rate of 1 s and 128 N/s, respectively, were recommended by the developers of the ADMIRE model (L. Forssell, personal communication, March 2004) and were used in this study. Figure 1a shows the corresponding AOA and pitch angle time history: The maximum AOA achieved was 16.0038 deg.

**Analysis 2**

In this analysis, pilot control inputs determined by the Clonk maneuver were applied, whereas the GA-based optimization algorithm was used to compute the worst-case combination of uncertain parameters. The results of this analysis are shown in Fig. 1b and Table 3. As expected, the effect of considering uncertainty in

**Table 3 Analyses 2, 4, and 5 results**

| Case | $\left[\Delta^*_{\text{mass}}\ \Delta^*_{x\,\text{cg}}\ \Delta^*_{C_{m_{\delta_e}}}\ \Delta^*_{I_{yy}}\ \Delta^*_{C_{m_\alpha}}\right]$ | Maximum $\alpha(t)$ |
|---|---|---|
| Analysis 2 | [0.0413 0.0750 0.0499 −0.1218 0.0500] | 28.6008 |
| Analysis 4 | [0.1000 0.0104 −0.0500 −0.2000 0.0429] | 61.7187 |
| Analysis 5 | [0.0767 0.0739 0.0400 0.1856 0.0500] | 69.36189 |

the aircraft simulation model has been to increase significantly the maximum value of AOA achieved.

**Analysis 3**

In this analysis, worst-case pilot control inputs are computed using the GA-based optimization algorithm for the nominal simulation model. The maximum AOA obtained from the analysis is 50.0233 deg. Figure 2 shows the corresponding pilot control inputs

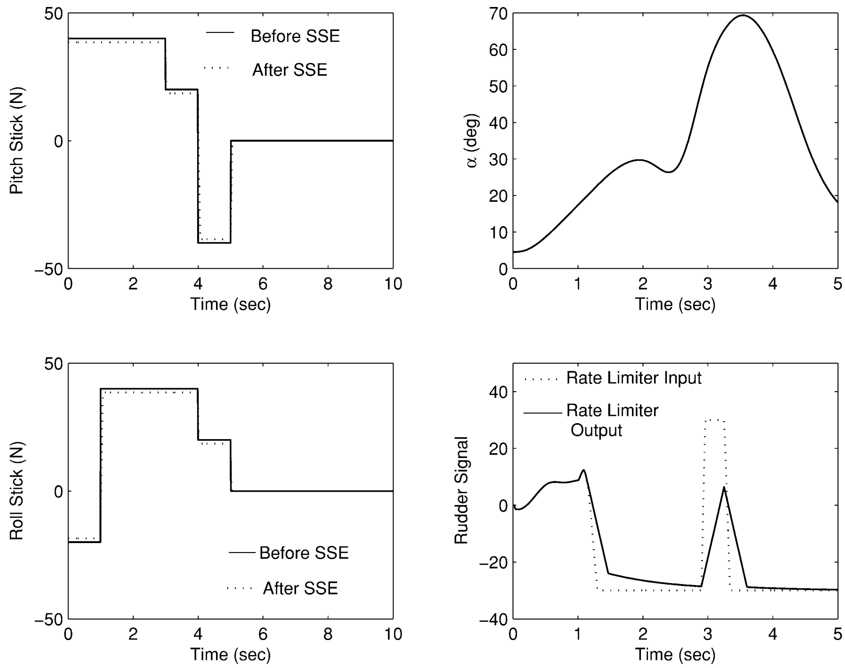



**Fig. 3 Analysis 5 results.**

and AOA time history. Note that the maximum AOA value is significantly higher than the one obtained by using the pilot inputs determined by the Clonk maneuver in both Analyses 1 and 2.

**Analysis 4**

In this analysis, the worst-case pilot control inputs generated by analysis 3 are used, whereas the GA-based optimization algorithm searches for the worst-case combination of uncertain parameters $\Delta$. This corresponds to searching for the worst-case pilot inputs and uncertain parameters separately. The uncertain parameter combination given in Table 3 gave the maximum value of AOA (61.7187 deg) for this approach. Figure 2 shows the corresponding AOA time history. Note that the worst-case values of two of the uncertain parameters are inside the hyperbox that defines the search space. Interestingly, if the value of $\Delta_{xcg}$ is changed to be at its maximum allowable value (which would correspond to the worst-case value most likely suggested by an intuitive interpretation of flight mechanics principles), the maximum AOA build up turns out to be only 48.7636 deg. This result shows the limitations of relying entirely on flight mechanics intuition when analysing highly nonlinear flight control problems. It also illustrates the limitations of current industrial approaches to identifying worst-case uncertain parameter combinations based on evaluating all combinations of minimum and maximum values of the parameters.[13]

**Analysis 5**

Analysis 4 has already shown the significant effect of the uncertain parameters on the maximum AOA value when a specific fixed pilot input is considered. In analysis 5, both the pilot control inputs and uncertain parameters $\Delta$ are optimized simultaneously. The solution from this analysis, therefore, has two parts: one the pilot control inputs and the other the value of the uncertain parameters. The results are shown in Fig. 3 and in Table 3. Figure 3 shows the pilot control input combination and the corresponding AOA time history. By the sixth second the aircraft responses are outside the available aerodynamic database, and hence, the simulation stops. The worst-case combination of uncertainties is given in Table 3. The maximum AOA overshoot obtained is 69.3543, which is significantly higher than that obtained in all of the preceding analyses. Notice also that (1) the worst-case pilot control input is very different from the one computed in the earlier analysis and (2), the worst-case values of all of the uncertain parameters are now located inside the search-space hyperbox. Finally, some explanation for the departure susceptibility of the aircraft for this particular sequence of pilot inputs can be found in the plot of the rudder actuator response shown in Fig. 3. The severe rate limiting apparent in the rudder actuator suggests the need for further improvement of the current lateral/directional control law and illustrates the contribution that this type of analysis can make to an iterative flight control law design cycle.

## VI. Conclusions

This paper has described an approach based on global optimization and nonlinear simulation that may be used as part of the process of clearing a flight control law against departure susceptibility and/or violations of envelope protection limits. The flexibility of global optimization methods is shown to allow for the simultaneous computation of worst-case pilot inputs and worst-case combinations of uncertain parameters in the nonlinear aircraft simulation model. The results show that only such a simultaneous consideration of worst-case pilot inputs and uncertain parameters is likely to reveal the true worst-case behavior of the aircraft. In addition, it was shown how global optimization methods can find very simple combinations of pilot input signals that have worse effects than the aggressive Clonk maneuver, which was developed via extensive piloted simulation trials.

## Acknowledgment

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